Overview of the Standards Chapters

of the

Mathematics Framework

for California Public Schools: Kindergarten Through Grade Twelve

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Overview of the Standards Chapters

These Standards are not intended to be new names for old ways of doing business.
—National Governors Association Center for Best Practices, Council of Chief State School Officers (NGA/CCSSO) 2010f

In 2009, the Council of Chief State School Officers (CCSSO) and the National Governors Association Center for Best Practices (NGA) committed to developing a set of standards that would help prepare students for success in careers and college. The Common Core State Standards Initiative was a voluntary, state-led effort coordinated by the CCSSO and NGA to establish clear and consistent education standards. Development of the standards began with research-based learning progressions detailing what is known about how students' mathematical knowledge, skills, and understanding develop over time.

In June 2010, the State of California replaced its existing mathematics standards by adopting the California Common Core State Standards for Mathematics (CA CCSSM). The state's previous mathematics standards had been in place since 1997. In January 2013, in accordance with Senate Bill 1200, the California State Board of Education (SBE) adopted modifications to the CA CCSSM, which included organizing the standards into model courses for higher mathematics aligned with Appendix A of the Common Core State Standards Initiative. Standards that are unique to California (California additions) are identified by boldface type and followed by the abbreviation **CA.**

California's new standards define what students should understand and be able to do in the study of mathematics. The state's implementation of the CA CCSSM demonstrates a continued commitment to providing a world-class education for all students that supports lifelong learning and the skills and knowledge necessary to participate in the global economy of the twenty-first century.

Understanding the California Common Core State Standards for Mathematics

The CA CCSSM were designed to help students gain proficiency with and understanding of mathematics across grade levels. The standards call for learning mathematical content in the context of real-world situations, using mathematics to solve problems, and developing "habits of mind" that foster mastery of mathematics content as well as mathematical understanding.

The standards for kindergarten through grade eight (K–8) prepare students for higher mathematics, beginning with Mathematics I or Algebra I, and serve as the foundation on which more advanced mathematical knowledge can be built. The standards for higher mathematics (high school–level standards) prepare students for college, careers, and productive citizenship. In short, the standards are a progression of mathematical learning.

The standards are based on three major principles: **focus, coherence, and rigor**. These principles are meant to fuel greater achievement in a rigorous curriculum, in which students acquire conceptual understanding, procedural skill and fluency, and the ability to apply mathematics to solve problems.

Major Principles of the California Common Core State Standards for Mathematics

- **Focus**—Place strong emphasis where the standards focus.
- **Coherence**—Think across grades, and link to major topics in each grade.
- **Rigor**—In major topics, pursue with equal intensity:
  - conceptual understanding;
  - procedural skill and fluency;
  - application.

Focus is necessary so that students have sufficient time to think about, practice, and integrate new ideas into their growing knowledge structure. Focus is also a way to allow time for the kinds of rich classroom discussion and interaction that support the Standards for Mathematical Practice (MP) and develop students' broader mathematical understanding. Instruction should focus deeply on only those concepts that are emphasized in the standards so that students can build a strong foundation in conceptual understanding, a high degree of procedural skill and fluency, and the ability to apply the mathematics they know to solve problems inside and outside the mathematics classroom.

Coherence arises from mathematical connections. Some of the connections in the standards knit topics together at a single grade level. Most connections are vertical, as the standards support a progression of increasing knowledge, skill, and sophistication across the grades.

- Thinking across grades: The standards are designed to help administrators and teachers connect learning within and across grades. For example, the standards develop fractions and multiplication across grade levels, so that students can build new understanding on foundations that were established in previous years. Thus each standard is an extension of previous learning, not a completely new concept.
- Linking to major topics: Connections between the standards at a single grade level can be used to improve the instructional focus by linking additional or supporting topics to the major work of the grade. For example, in grade three, bar graphs are not "just another topic to cover." Students use information presented in bar graphs to solve word problems using the four operations of arithmetic. (For lists of Major and Additional/Supporting topics, see the Cluster-Level Emphases charts in each grade-level chapter.)

Grades	Priorities in Support of Rich Instruction: Expectations of Fluency and Conceptual Understanding in the CA CCSSM
K–2	Addition and subtraction—concepts, skills, problem solving, and place value
3–5	Multiplication and division of whole numbers and fractions—concepts, skills, and problem solving
6	Ratios and proportional reasoning; early expressions and equations
7	Ratios and proportional reasoning; arithmetic of rational numbers
8	Linear algebra

Adapted from Achieve the Core 2012.

Rigor requires that conceptual understanding, procedural skill and fluency, and application be approached with equal intensity.

- Conceptual understanding: The word *understand* is used in the standards to set explicit expectations for conceptual understanding. Teachers focus on much more than "how to get the answer"; they support students' ability to access concepts from a number of different perspectives. Students might demonstrate deep conceptual understanding of core mathematics concepts by solving short conceptual problems, applying mathematics in new situations, and speaking and writing about their understanding. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, such students may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, help other students understand a given method or find and correct an error, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices.

Examples of Understanding in the CA CCSSM	
Grade/Level	**Standards**
K	**Understand** that each successive number name refers to a quantity that is one larger (**K.CC.4c**).
2	**Understand** that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds (**2.NBT.7**).
4	**Understand** addition and subtraction of fractions as joining and separating parts referring to the same whole (**4.NF.3a**).
6	**Understand** the concept of a ratio and use ratio language to describe a ratio relationship between two quantities (**6.RP.1**).
8	**Understand** that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output (**8.F.1**).
Higher Mathematics	**Understand** that a function from one set (called the *domain*) to another set (called the *range*) assigns to each element of the domain exactly one element of the range (**F-IF.1**). (*Note:* This is only a portion of the complete standard.)
Higher Mathematics	Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles (**G-SRT.6**).

- Procedural skill and fluency: The standards are explicit where fluency is expected. In kindergarten through grade six (K–6), students should make steady progress toward procedural skill and computational fluency (being accurate and reasonably fast), including knowing single-digit products and sums from memory (for example, see **2.OA.2** and **3.OA.7**). As used in the standards, the word *fluently* refers to fluency with a written or mental method, not a method using manipulatives or concrete representations. Progress toward fluency should be woven into instruction in grade-appropriate ways, along with developing conceptual understanding of the four operations.[1]

1 . For more information about how students develop fluency in tandem with understanding, see the University of Arizona's Progressions documents on Operations and Algebraic Thinking and on Number and Operations in Base Ten (the University of Arizona Progressions Documents for the Common Core Math Standards [UA Progressions] 2011–13).

Manipulatives and concrete representations such as diagrams that enhance conceptual understanding can help students make connections to written and symbolic methods (e.g., see **1.NBT.1**). Methods and algorithms should be general and based on principles of mathematics (e.g., place value and properties of operations).

Developing fluency with single-digit computations can involve a mixture of just knowing some answers, knowing some answers from understanding patterns, and knowing some answers from understanding and using strategies. In grades four, five, and six, moving to fluency with multi-digit computations and operations with decimals and fractions requires developing a base of understanding in previous years about how to use place value in carrying out and interpreting operations with single digits within a multi-digit number and understanding how to use unit fractions and equivalence for meaningful fraction operations. Students examine various methods and relate them to visual models, but from the beginning students develop, discuss, and use efficient, accurate, and generalizable methods that are or will lead to a variation of the standard algorithm. Students drop the visual models when they can, although they may continue to use models if needed. *Fluency* means working without visual models. Sufficient practice and extra support should be provided at each grade to allow all students to meet the standards that explicitly call for fluency.

Grade	Examples of Expectations of Fluency in the K–6 CA CCSSM
K	Add/subtract within 5
1	Add/subtract within 10
2	Add/subtract within 20 (using mental strategies) Add/subtract within 100 (using strategies[2])
3	Multiply/divide within 100 Add/subtract within 1,000 (using algorithms[3])
4	Add/subtract whole numbers within 1,000,000 (using the standard algorithm[4])
5	Multiply multi-digit numbers (using the standard algorithm) Add/subtract fractions
6	Divide multi-digit numbers (using the standard algorithm) Perform multi-digit decimal operations (add, subtract, multiply, and divide using the standard algorithm for each operation)

Adapted from Achieve the Core 2012.

2. These strategies would be based on place value, properties of operations, and/or the relationship between addition and subtraction.

3. A range of algorithms may be used.

4. Minor variations of writing the standard algorithm are acceptable.

- Application: Students are expected to use mathematics to solve "real-world problems." In the standards, the phrase *real-world problems* and the star symbol (★) are used to set expectations and flag opportunities for applications and modeling (which is a Standard for Mathematical Practice as well as a Conceptual Category in higher mathematics). Real-world problems and standards that support modeling are also opportunities to provide activities related to careers and everyday life. Teachers in content areas outside of mathematics—particularly science—ensure that students use mathematics at all grade levels to make meaning of and access content (adapted from Achieve the Core 2012).

Progression to Higher Mathematics

The progression from kindergarten standards to standards for higher mathematics, beginning with Mathematics I or Algebra I, exemplifies the three principles of focus, coherence, and rigor that are central to the CA CCSSM.

In kindergarten through grade five (K–5), the focus is on addition, subtraction, multiplication, and division of whole numbers, fractions, and decimals, with a balance of concepts, skills, and problem solving. Arithmetic is viewed as an important set of skills and also as a thinking subject that prepares students for higher mathematics. Measurement and geometry develop alongside number and operations and are tied specifically to arithmetic along the way.

In middle school, multiplication and division develop into the powerful forms of ratio and proportional reasoning. The properties of operations take on prominence as arithmetic matures into algebra. The theme of quantitative relationships also becomes explicit in grades six through eight, developing into the formal concept of a function by grade eight. Meanwhile, the foundations of deductive geometry are laid in the middle grades. Finally, the gradual development of data representations in kindergarten through grade five leads to statistics in middle school: the study of shape, center, and spread of data distributions; possible associations between two variables; and the use of sampling in making statistical decisions.

In higher mathematics, algebra, functions, geometry, and statistics develop with an emphasis on modeling. Students continue to take a thinking approach to algebra, learning to see and make use of structure in algebraic expressions of growing complexity (Partnership for Assessment of Readiness for College and Careers [PARCC] 2012).

Mathematics is a logically progressing discipline that has intricate connections among the various domains and clusters in the standards. Sustained practice is required to master grade-level and course-level content. The major work (or emphases) in the standards for kindergarten through grade eight is noted in the Cluster-Level Emphases charts presented in each of the grade-level chapters that follow. Further, table OV-1 (adapted from Achieve the Core 2012) summarizes an important subset of the major work in kindergarten through grade eight, as the progression of learning in the standards leads toward Mathematics I or Algebra I.

Table OV-1. Progression to Algebra I and Mathematics I in Kindergarten Through Grade Eight

Kindergarten	Grade One	Grade Two	Grade Three	Grade Four	Grade Five	Grade Six	Grade Seven	Grade Eight
Know number names and the count sequence Count to tell the number of objects Compare numbers Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from Work with numbers 11–19 to gain foundations for place value	Represent and solve problems involving addition and subtraction Understand and apply properties of operations and the relationship between addition and subtraction Add and subtract within 20 Work with addition and subtraction equations Extend the counting sequence Understand place value Use place-value understanding and properties of operations to add and subtract Measure lengths indirectly and by iterating length units	Represent and solve problems involving addition and subtraction Add and subtract within 20 Understand place value Use place-value understanding and properties of operations to add and subtract Measure and estimate lengths in standard units Relate addition and subtraction to length	Represent and solve problems involving multiplication and division Understand properties of multiplication and the relationship between multiplication and division Multiply and divide within 100 Solve problems involving the four operations, and identify and explain patterns in arithmetic Develop understanding of fractions as numbers Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects Geometric measurement: understand concepts of area, and relate area to multiplication and to addition	Use the four operations with whole numbers to solve problems Generalize place-value understanding for multi-digit whole numbers Use place-value understanding and properties of operations to perform multi-digit arithmetic Extend understanding of fraction equivalence and ordering Build fractions from unit fractions by applying and extending previous understandings of operations Understand decimal notation for fractions, and compare decimal fractions	Understand the place-value system Perform operations with multi-digit whole numbers and decimals to hundredths Use equivalent fractions as a strategy to add and subtract fractions Apply and extend previous understandings of multiplication and division to multiply and divide fractions Geometric measurement: understand concepts of volume, and relate volume to multiplication and to addition Graph points in the coordinate plane to solve real-world and mathematical problems*	Apply and extend previous understandings of multiplication and division to divide fractions by fractions Apply and extend previous understandings of numbers to the system of rational numbers Understand ratio concepts and use ratio reasoning to solve problems Apply and extend previous understandings of arithmetic to algebraic expressions Reason about and solve one-variable equations and inequalities Represent and analyze quantitative relationships between dependent and independent variables	Apply and extend previous understanding of operations with fractions to add, subtract, multiply, and divide rational numbers Analyze proportional relationships and use them to solve real-world and mathematical problems Use properties of operations to generate equivalent expressions Solve real-life and mathematical problems using numerical and algebraic expressions and equations	Work with radicals and integer exponents Understand the connections between proportional relationships, lines, and linear equations Analyze and solve linear equations and pairs of simultaneous linear equations Define, evaluate, and compare functions Use functions to model relationships between quantities

Adapted from Achieve the Core 2012.

*Indicates a cluster that is well thought of as part of a student's progress to algebra, but that is currently not designated as Major by one or both of the assessment consortia (PARCC and Smarter Balanced) in their draft materials. Apart from the one exception marked by an asterisk, the clusters listed here are a subset of those designated as Major in both of the assessment consortia's draft documents.

Two Types of Standards

The CA CCSSM include two types of standards: Standards for Mathematical Practice and Standards for Mathematical Content. These standards address "habits of mind" that students should develop to foster mathematical understanding and expertise, as well as concepts, skills, and knowledge—what students need to understand, know, and be able to do. The standards also require that mathematical practices and mathematical content be connected. These connections are essential to support the development of students' broader mathematical understanding, as students who lack understanding of a topic may rely too heavily on procedures. The Standards for Mathematical Practice must be taught as carefully and practiced as intentionally as the Standards for Mathematical Content are. Neither type should be isolated from the other; mathematics instruction is most effective when these two aspects of the CA CCSSM come together as a powerful whole.

The eight **Standards for Mathematical Practice** (**MP**) describe the attributes of mathematically proficient students and expertise that mathematics educators at all levels should seek to develop in their students; see table OV-2. Mathematical practices provide a vehicle through which students engage with and learn mathematics. As students move from elementary school through high school, mathematical practices are integrated in the tasks as students engage in doing mathematics and master new and more advanced mathematical ideas and understandings.

Standards for Mathematical Practice (**MP**)
These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the National Council of Teachers of Mathematics' process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding, procedural fluency, and productive disposition (NGA/CCSSO 2010q).

Table OV-2. Standards for Mathematical Practice (MP)

MP.1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

MP.2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of the quantities and their relationships in problem situations. Students bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically, and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize,* to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meanings of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

MP.3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen to or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments. **Students build proofs by induction and proofs by contradiction. CA.3.1** (for higher mathematics only).

Table OV-2 *(continued)*

MP.4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts, and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

MP.5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a Web site, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

MP.6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, expressing numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school, they have learned to examine claims and make explicit use of definitions.

MP.7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square, and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Table OV-2 *(continued)*
MP.8 Look for and express regularity in repeated reasoning.
Mathematically proficient students notice if calculations are repeated and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Table OV-3 summarizes the eight MP standards and provides examples of questions that teachers might use to support mathematical thinking and student engagement (as called for in the MP standards).

Table OV-3

Summary of the Standards for Mathematical Practice	Questions to Develop Mathematical Thinking
MP.1 Make sense of problems and persevere in solving them. • Mathematically proficient students interpret and make meaning of the problem to find a starting point. • Analyze what is given in order to explain to themselves the meaning of the problem. • Plan a solution pathway instead of jumping to a solution. • Monitor their own progress and change the approach if necessary. • See relationships between various representations. • Relate current situations to concepts or skills previously learned and connect mathematical ideas to one another. • Continually ask themselves, “Does this make sense?” • Can understand various approaches to solutions.	• How would you describe the problems in your own words? • How would you describe what you are trying to find? • What do you notice about ________? • What information is given in the problem? • Describe the relationship between the quantities. • Describe what you have already tried. What might you change? • Talk me through the steps you have used to this point. • What steps in the process are you most confident about? • What are some other strategies you might try? • What are some other problems that are similar to this one? • How might you use one of your previous problems to help you begin? • How else might you [organize, represent, show, etc.] ________?

Table OV-3 *(continued)*	
Summary of the Standards for Mathematical Practice	**Questions to Develop Mathematical Thinking**
MP.2 Reason abstractly and quantitatively. • Mathematically proficient students make sense of quantities, and the relationships between quantities, in problem situations. • Decontextualize (represent a situation symbolically and manipulate the symbols) and contextualize (make meaning of the symbols in a problem) quantitative relationships. • Understand the meaning of quantities and flexibly use operations and their properties. • Create a logical representation of the problem. • Attend to the meaning of quantities, not just how to compute them.	• What do the numbers used in the problem represent? • What is the relationship of the quantities? • How is _________ related to _________? • What is the relationship between _________ and _________? • What does _________ mean to you? (e.g. symbol, quantity, diagram) • What properties might we use to find a solution? • How did you decide that you needed to use _________ in this task? • Could we have used another operation or property to solve this task? Why or why not?
MP.3 Construct viable arguments and critique the reasoning of others. • Mathematically proficient students analyze problems and use stated mathematical assumptions, definitions, and established results in constructing arguments. • Justify conclusions with mathematical ideas. • Listen to the arguments of others, and ask useful questions to determine if an argument makes sense. • Ask clarifying questions or suggest ideas to improve or revise the argument. • Compare two arguments and determine if the logic is correct or flawed.	• What mathematical evidence would support your solution? • How can we be sure that _________? How could you prove that _________? • Will it still work if _________? • What were you considering when _________? • How did you decide to try that strategy? • How did you test whether your approach worked? • How did you decide what the problem was asking you to find? (What was unknown?) • Did you try a method that did not work? Why didn't it work? Would it ever work? Why or why not? • What is the same and what is different about _________? • How could you demonstrate a counter-example? • I think it might be clearer if you said _________. Is that what you meant? • Is your method like Shawna's method? If not, how is your method different?

Table OV-3 *(continued)*

Summary of the Standards for Mathematical Practice	Questions to Develop Mathematical Thinking
MP.4 Model with mathematics. • Mathematically proficient students understand this is a way to reason quantitatively and abstractly (able to decontextualize and contextualize). • Apply the mathematics they know to solve everyday problems. • Simplify a complex problem and identify important quantities to look at relationships. • Represent mathematics to describe a situation either with an equation or a diagram, and interpret the results of a mathematical situation. • Reflect on whether the results make sense, possibly improving or revising the model. • Ask themselves, "How can I represent this mathematically?"	• What math drawing or diagram could you make and label to represent the problem? • What are some ways to represent the quantities? • What is an equation or expression that matches the [diagram, number line, chart, table, etc.]? • Where did you see one of the quantities in the task in your equation or expression? • How would it help to create a [diagram, graph, table, etc.]? • What are some ways to visually represent _________? • What formula might apply in this situation?
MP.5 Use appropriate tools strategically. • Mathematically proficient students use available tools including visual models, recognizing the strengths and limitations of each. • Use estimation and other mathematical knowledge to detect possible errors. • Identify relevant external mathematical resources to pose and solve problems. • Use technological tools to deepen their understanding of mathematics.	• What mathematical tools could we use to visualize and represent the situation? • What information do you have? • What do you know that is not stated in the problem? • What approach would you consider trying first? • What estimate did you make for the solution? • In this situation, would it be helpful to use a [graph, number line, ruler, diagram, calculator, manipulatives, etc.]? • Why was it helpful to use _________? • What can using a _________ show us that _________ may not? • In what situations might it be more informative or helpful to use _________?

Table OV-3 *(continued)*	
Summary of the Standards for Mathematical Practice	**Questions to Develop Mathematical Thinking**
MP.6 Attend to precision. • Mathematically proficient students communicate precisely with others and try to use clear mathematical language when discussing their reasoning. • Understand the meanings of symbols used in mathematics and can label quantities appropriately. • Express numerical answers with a degree of precision appropriate for the problem context. • Calculate efficiently and accurately.	• What mathematical terms apply in this situation? • How did you know your solution was reasonable? • Explain how you might show that your solution answers the problem. • What would be a more efficient strategy? • How are you showing the meaning of the quantities? • What symbols or mathematical notations are important in this problem? • What mathematical language, definitions, properties (and so forth) can you use to explain ________? • Can you say it in a different way? • Can you say it in your own words? And now say it in mathematical words. • How could you test your solution to see if it answers the problem?
MP.7 Look for and make use of structure. • Mathematically proficient students look for the overall structures and patterns in mathematics and think about how to describe these in words, mathematical symbols, or visual models. • See complicated things as single objects or as being composed of several objects. Compose and decompose conceptually. • Apply general mathematical patterns, rules, or procedures to specific situations.	• What observations can you make about ________? • What do you notice when ________? • What parts of the problem might you [eliminate, simplify, etc.]? • What patterns do you find in ________? • How do you know if something is a pattern? • What ideas that we have learned before were useful in solving this problem? • What are some other problems that are similar to this one? • How does this relate to ________? • In what ways does this problem connect to other mathematical concepts?

Table OV-3 *(continued)*

Summary of the Standards for Mathematical Practice	Questions to Develop Mathematical Thinking
MP.8 Look for and express regularity in repeated reasoning. • Mathematically proficient students see repeated calculations and look for generalizations and shortcuts. • See the overall process of the problem and still attend to the details in the problem-solving steps. • Understand the broader application of patterns and see the structure in similar situations. • Continually evaluate the reasonableness of their intermediate results.	• Explain how this strategy works in other situations. • Is this always true, sometimes true, or never true? • How would we prove that ________? • What do you notice about ________? • What is happening in this situation? • What would happen if ________? • Is there a mathematical rule for ________? • What predictions or generalizations can this pattern support? • What mathematical consistencies do you notice? • How is this situation like and different from other situations using this operation?

Adapted from Kansas Association of Teachers of Mathematics 2012, 3rd Grade Flipbook.

Ideally, several MP standards will be evident in each lesson as they interact and overlap with each other. The MP standards are not a checklist; they are the basis for mathematics instruction and learning. To help students persevere in solving problems (**MP.1**), teachers need to allow their students to struggle productively, and they must be attentive to the type of feedback they provide to students. Dr. Carol Dweck's research (Dweck 2006) revealed that feedback offering praise of effort and perseverance seems to engender and reinforce a "growth mindset."[5] In Dweck's estimation, "[g]rowth-minded teachers tell students the truth [about being able to close the learning gap between them and their peers] and then give them the tools to close the gap" (Dweck 2006).

Structuring the MP standards can help educators recognize opportunities for students to engage with mathematics in grade-appropriate ways. In figure OV-1, the eight MP standards are grouped into four categories. These four pairs of standards can also be given names, beginning with the rectangle on the far left and then moving from the bottom to the top with the other three rectangles. These names can become a sentence teachers might ask at the end of every day—for example, "Did I *Make Sense of Math* and *Math Structure* by using *Math Drawings* to support *Math Reasoning*?" This approach can help teachers to continually incorporate the core of the MP standards into classroom practices.

5. According to Dweck, a person with a growth mindset believes that intelligence is something that can be nurtured and gained. When people with this type of mindset do not meet the expected level of performance on a test or an assignment or have difficulty understanding a concept, they work hard at it, believing that if they just try hard enough, they will achieve the desired outcome.

Figure OV-1. Structuring the Standards for Mathematical Practice (MP)

Overarching habits of mind of a productive mathematical thinker		
1. Make sense of problems and persevere in solving them. 6. Attend to precision.	2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others.	Reasoning and explaining
	4. Model with mathematics. 5. Use appropriate tools strategically.	Modeling and using tools
	7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.	Seeing structure and generalizing

Source: McCallum 2011.

The **Standards for Mathematical Content** were built on progressions of topics across a number of grade levels, informed both by research on children's cognitive development and by the logical structure of mathematics.

Kindergarten Through Grade Eight

In kindergarten through grade eight, the standards are organized by grade level and then by **domains** (clusters of standards that address "big ideas" and support connections of topics across the grades), **clusters** (groups of related standards inside domains), and finally by the **standards** (what students should understand and be able to do). The standards do not dictate curriculum or pedagogy. For example, just because Topic A appears before Topic B in the standards for a given grade, it does not mean that Topic A must be taught before Topic B (NGA/CCSSO 2010c).

Throughout this framework, specific standards or groups of standards are identified in the narrative. For example, as shown in figure OV-2, a narrative reference to **3.NBT.1–3** signifies a standard at the third-grade level, the domain Number and Operations in Base Ten (**NBT**), and standards 1, 2, and 3 in the first cluster.

Figure OV-2. How to Read the Standards for Kindergarten Through Grade Eight

Domain — Grade Level — Domain Abbreviation

Number and Operations in Base Ten **3.NBT**

Use place-value understanding and properties of operations to perform multi-digit arithmetic.

1. Use place-value understanding to round whole numbers to the nearest 10 or 100.
2. Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.
3. Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9 × 80, 5 × 60) using strategies based on place value and properties of operations.

Standard — **Cluster**

Higher Mathematics

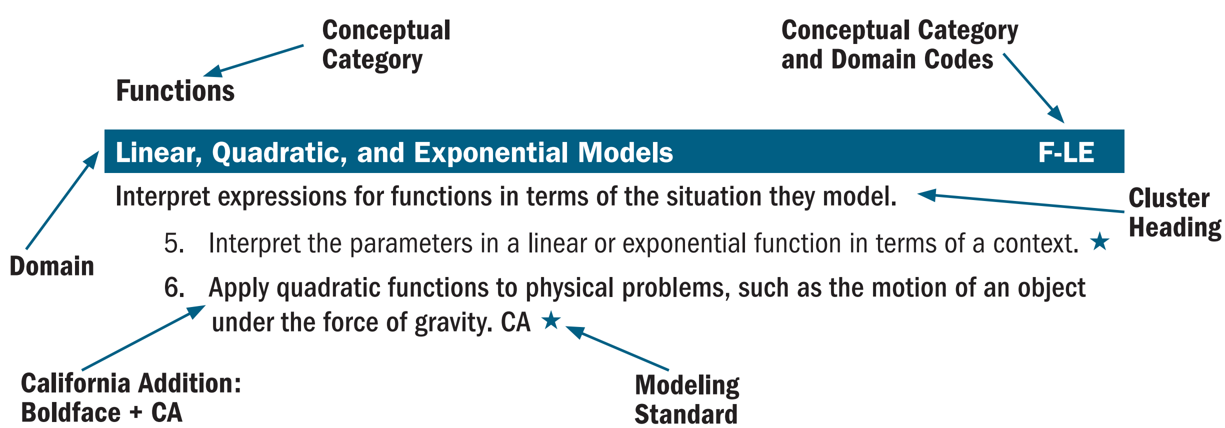
The standards for higher mathematics are organized differently than the K–8 standards. When developed by the NGA/CCSSO, the higher mathematics standards were not organized into courses; instead, they were listed according to the following conceptual categories:

- Number and Quantity (N)
- Algebra (A)
- Functions (F)
- Modeling (★)
- Geometry (G)
- Statistics and Probability (S)

Conceptual categories present a coherent view of higher mathematics; a student's work with functions, for example, crosses a number of traditional course boundaries, potentially up through and including calculus. With the exception of Modeling (see explanation following figure OV-3), each conceptual category is further subdivided into several domains, and each domain is subdivided into clusters. This structure is similar to that of the grade-level content standards.

Each higher mathematics standard begins with the identifier for the conceptual category (N, A, F, G, S), followed by the domain code, and then the standard number.

Figure OV-3. How to Read the Standards for Higher Mathematics



The two standards in figure OV-3 would be referred to as **F-LE.5** and **F-LE.6**, respectively. The star symbol (★) indicates that both standards are also Modeling standards. Modeling is best interpreted not as a collection of isolated topics, but rather in relation to other standards. Readers are encouraged to refer to appendix B for an extensive explanation of the Modeling conceptual category.

Table OV-4 illustrates how the domains and conceptual categories are distributed across the K–12 mathematical content standards. The corresponding abbreviations for each are also identified—for example, Geometry (G).

Table OV-4. Mathematical Content Domains (K–8) and Conceptual Categories (Higher Mathematics)

<table>
<tr><th>Grade</th><th>K</th><th>1</th><th>2</th><th>3</th><th>4</th><th>5</th><th>6</th><th>7</th><th>8</th><th colspan="2">Higher Mathematics Conceptual Categories</th></tr>
<tr><td rowspan="6">K–8 Domains</td><td>Counting and Cardinality (CC)</td><td></td><td></td><td></td><td></td><td></td><td colspan="2">Ratios and Proportional Relationships(RP)</td><td>Functions (F)</td><td>Functions (F)</td><td rowspan="6">Modeling (★)</td></tr>
<tr><td colspan="6">Operations and Algebraic Thinking (OA)</td><td colspan="3">Expression and Equations (EE)</td><td>Algebra (A)</td></tr>
<tr><td colspan="6">Number and Operations in Base Ten (NBT)</td><td colspan="3" rowspan="2">The Number System (NS)</td><td rowspan="2">Number and Quantity (N)</td></tr>
<tr><td></td><td></td><td></td><td colspan="3">Number and Operations—Fractions (NF)</td></tr>
<tr><td colspan="6">Measurement and Data (MD)</td><td colspan="3">Statistics and Probability (SP)</td><td>Statistics and Probability (S)</td></tr>
<tr><td colspan="6">Geometry (G)</td><td colspan="3">Geometry (G)</td><td>Geometry (G)</td></tr>
</table>

Overview: K–8 Chapters

The chapters covering kindergarten through grade eight provide guidance on instruction and learning aligned with the CA CCSSM. Each chapter presents a brief summary of prior learning and an overview of what students learn at that grade level. A section on the Standards for Mathematical Content highlights the instructional focus of the standards at the grade and includes a Cluster-Level Emphases chart that designates clusters of standards as “Major” or “Additional/Supporting” work at the grade level. The Connecting Mathematical Practices and Content section provides grade-level explanations and examples of how the MP standards may be integrated into grade-level-appropriate tasks.

The largest section of each chapter is a description of Standards-Based Learning organized by domains and clusters, with exemplars to explain the content standards, highlight connections to the various mathematical practice standards, and demonstrate the importance of developing conceptual understanding, procedural skill and fluency, and application. Also noted are opportunities to link concepts in the Additional/Supporting clusters to Major work at the grade (based on the grade-specific Cluster-Level Emphases charts) and examples of focus, coherence, and rigor. Finally, each chapter presents “Essential Learning for the Next Grade” to highlight important knowledge, skills, and understanding that students will need to succeed in future grades. The grade-level content standards are embedded throughout the narrative and at the end of each chapter. Standards that are unique to California (California additions) are identified by boldface type and followed by the abbreviation **CA**.

Overview: Higher Mathematics Chapters

When first adopted in August 2010, the CA CCSSM for higher mathematics were organized differently than the K–8 standards—by conceptual categories rather than in courses. In January 2013, the SBE adopted modifications to the CA CCSSM, including organizing content standards into model courses for higher mathematics, in accordance with Senate Bill 1200 (*Education Code* Section 60605.11, Chapter 654, Statues of 2012).

The model courses are organized into two pathways: Traditional and Integrated. The framework includes a description of these courses. The content of these courses is the same, regardless of the grade level at which they are taught.

Standards for Mathematical Practice

The MP standards are interwoven throughout the higher mathematics curriculum. Instruction should focus equally on developing students' ability to engage in the practice standards and on developing conceptual understanding of and procedural fluency in the content standards. The MP standards are the same at each grade level, with the exception of an additional practice standard included only in the CA CCSSM for higher mathematics:

> **MP.3.1 CA: Students build proofs by induction and proofs by contradiction.**
>
> This standard can be seen as an extension of Mathematical Practice 3, in which students construct viable arguments and critique the reasoning of others.

In the higher mathematics courses, the levels of sophistication of each MP standard increase as students integrate grade-appropriate mathematical practices with the content standards. Examples of the MP standards appear in each higher mathematics course narrative.

Standards for Mathematical Content

The entire catalog of higher mathematics standards is presented in the *California Common Core State Standards: Mathematics* (CDE 2013a), organized by both model courses and conceptual category. In this framework, the standards are organized into model courses that were adopted by the SBE in January 2013. The higher mathematics content standards specify the mathematics that all students should study in order to be college- and career-ready. Additional mathematical content that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics is indicated by a (+) symbol, as in this example:

4. (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.

All standards without a (+) symbol should be included in the common mathematics curriculum for all college- and career-ready students. Standards with a (+) symbol may also appear in courses intended for all students.

Higher Mathematics Chapters

The higher mathematics chapters are organized into courses according to two pathways:

- **Traditional Pathway** — consists of the higher mathematics standards organized along more traditional lines into Algebra I, Geometry, and Algebra II courses. In this sequence, almost the entire Geometry conceptual category is separated into a single course and treated as a separate subject. Although these courses have the same names as their traditional counterparts, it is important to note that the nature of the CA CCSSM yields very different courses. In the past, the label "Geometry" referred to a specific course, but now it may also refer to the conceptual category. Care will be taken throughout the higher mathematics chapters to make the distinction clear.
- **Integrated Pathway** — consists of the courses Mathematics I, II, and III. The integrated pathway presents higher mathematics as a connected subject, in that each course contains standards from all six of the conceptual categories. For example, in Mathematics I, students will focus on linear functions. Students contrast linear functions with exponential functions, solve linear equations, and model with functions. They also investigate the geometric properties of graphs of linear functions (lines) and model statistical data with lines of best fit. This is the way in which most other high-performing countries present higher mathematics, and it maintains the theme developed in kindergarten through grade eight of mathematics being a connected, multifaceted subject.

As noted earlier, regardless of the grade level at which a course is taught, the content of these courses is the same; for example, an Algebra I course or Mathematics I course is aligned with the Algebra I or Mathematics I course presented in the higher mathematics chapters of the framework. This is also true for advanced courses mentioned below.

In addition, the framework contains suggested courses in Precalculus and Statistics and Probability composed of CA CCSSM and an appendix on Mathematical Modeling (see appendix B). The Precalculus course mainly consists of standards with a (+) symbol, about two-thirds of which have not yet been taught in either the Integrated or Traditional Pathway; the course is designed to provide appropriate preparation for Calculus. The 1997 Calculus and Advanced Placement Probability and Statistics courses are also included.

Local educational agencies are not limited to offering the higher mathematics courses described in this framework. Beyond providing the courses necessary for students to fulfill the state requirements for high school graduation, local districts make decisions about which courses to offer their students. For example, career technical education (CTE) courses that integrate the higher mathematics CA CCSSM with technical and work-related knowledge and skills can make mathematics more relevant to students and can be an alternate yet rigorous pathway which prepares students for technical education programs after high school. CTE courses provide opportunities for students to engage in hands-on activities, problem solving, and decision making while learning in an occupational setting. The *California Career Technical Education Model Curriculum Standards* are a vital resource for designing CTE courses that incorporate the CA CCSSM.[6] There are also CTE courses developed by groups of educators at the University of California Curriculum Integration (UCCI) Institutes that balance academic rigor with career technical content and meet the mathematics component of the A–G requirements for college admission.[7] In addition, appendix B provides guidance to assist local educational agencies in designing a higher mathematics course in modeling.

The *Statement on Competencies in Mathematics Expected of Entering College Students,* issued by the Intersegmental Committee of the Academic Senates of the California Community Colleges, the California State University, and the University of California (ICAS 2013), is another document that local educational agencies may want to consult as they determine which courses to offer and what content to incorporate into the courses. This document describes the characteristics, skills, and knowledge students need in order to succeed in college.

Each CA CCSSM course is described in its own chapter, starting with an overview of the course followed by a detailed description of the mathematics content standards that are included in the course. Throughout, there are examples that illustrate the mathematical ideas and connect the MP standards to the content standards. In particular, standards that are expected to be new to existing secondary teachers are explained more fully than standards that have appeared in the curriculum prior to the adoption of the CA CCSSM.

It is important to note that some CA CCSSM standards are broad in scope and, as a result, are included in more than one course. When this occurs, a parenthetical comment is included with the standard to clarify the intent of the standard for that course. For example, the following standard appears in both Algebra I and Algebra II and has a different parenthetical comment for each course:

Algebra I

Arithmetic with Polynomials and Rational Expressions **A-APR**

Perform arithmetic operations on polynomials. [Linear and quadratic]

1. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

Algebra II

Arithmetic with Polynomials and Rational Expressions **A-APR**

Perform arithmetic operations on polynomials. [Beyond quadratic]

1. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

6. California's CTE model curriculum standards are viewable at http://www.cde.ca.gov/ci/ct/sf/ctemcstandards.asp (accessed April 9, 2014).

7. For additional information, go to ~~http://www.ucop.edu/agguide/career-technical-education/index.html~~ (accessed April 9, 2014).

In Algebra I, the notation specifies that the standard applies to linear and quadratic expressions. In Algebra II, the notation specifies that the standard applies to all expressions beyond quadratic.

California's new mathematics framework is a vital document that teachers will reference often; it is not a publication offering "business as usual." This framework embodies the belief that all students can learn mathematics and contains essential information for teachers and other stakeholders about universal access to the curriculum, teaching strategies, assessment, technology, modeling, and instructional materials. The framework also provides school and district administrators with information about how to support high-quality instruction. It is important for teachers of a single grade level to read not only their respective grade-level or course chapter in the framework, but also the grade level or chapter immediately preceding *and* following their particular area of focus. This will help teachers to plan a coherent, focused, and rigorous course of study.